

# REPORT ON GRG18, SESSION A3 MATHEMATICAL STUDIES OF THE FIELD EQUATIONS

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ABSTRACT. In this report I will give a summary of some of the main topics covered in Session A3, mathematical studies of the field equations, at GRG18, Sydney. Unfortunately, due to length constraints, some of the topics covered at the session will be very briefly mentioned or left out altogether. The summary is mainly based on extended abstracts submitted by the speakers and some of those who presented posters at the session. I would like to thank all participants for their contributions and help with this summary.

## 1. THE BUCHDAHL INEQUALITY

The Buchdahl inequality, which is included in most textbooks on general relativity, states that for a static, self-gravitating body,

$$(1) \quad 2M/R \leq 8/9,$$

where  $M$  is the ADM mass and  $R$  the area radius of the boundary of the static body. The proof by Buchdahl, cf. [16] of (1) assumed that the energy density is non-increasing outwards and that the pressure is isotropic. A bound on  $2M/R$  has an immediate observational consequence: if  $2M/R < 8/9$  then the gravitational red shift is less than 2 but if  $2M/R$  approaches 1 the red shift is unbounded. The assumptions used to derive the inequality are very restrictive, and as e.g. pointed out by Guven and Ó Murchada [27] neither of them hold in a simple soap bubble and they do not approximate any known topologically stable field configuration.

In addition to the restrictions implied by the hypotheses made by Buchdahl there are two other disadvantages associated with this inequality: it refers to the boundary of the body (i.e. the interior is excluded) and the solution which gives equality in the inequality is the Schwarzschild interior solution which has constant energy density and for which the pressure blows up at the centre. In particular it violates the dominant energy condition.

**Håkan Andréasson** explained recent work [2], where all four restrictions described above are eliminated. He considered matter models for which the energy density  $\rho$  and the radial pressure  $p$  is non-negative and which satisfies  $p + 2p_T \leq \Omega\rho$ , where  $\Omega$  is a non-negative constant and  $p_T$  is the tangential pressure, and showed that

$$(2) \quad \sup_{r>0} \frac{2m(r)}{r} \leq \frac{(1 + 2\Omega)^2 - 1}{(1 + 2\Omega)^2}.$$

Here  $m$  is the quasi-local mass so that  $M = m(R)$ . The case  $\Omega = 1$  gives the bound  $2m/r \leq 8/9$ . Among the matter models which satisfy the conditions stated are Vlasov matter and matter which satisfies the dominant energy condition and has

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non-negative pressure. Inequality (2) is sharp in the sense of measures and there are examples which come arbitrarily close to saturating the inequality. Andréasson has recently generalized the inequality to the case of charged spheres [3].

## 2. INITIAL-BOUNDARY VALUE PROBLEMS

A long standing problem in the analytical and numerical study of the Einstein equations is the choice of boundary conditions for boundaries near “infinity”. As is well known, the Einstein equations in harmonic coordinates reduces to a quasilinear system of wave equations. It was proved in previous work by Kreiss and Winicour cf. [36] that the initial-boundary value problem is well posed for such systems with the Sommerfeld outgoing boundary condition. The proof relied on pseudo-differential operator techniques.

**Oscar Reula** reported on a recent proof [35] of this fact which makes use only of partial integrations. The proof is much more transparent than the previous proof, and the argument based on partial integration is well adapted to the analysis of discretizations of the wave equation based on difference operators which have good “partial summation” properties.

Let  $(M, g_{ab})$  be a spacetime with timelike boundary  $\mathcal{T}$ . Let  $M$  be foliated by spacelike surfaces  $\Sigma_t$ , with timelike unit normal  $n^a$ . Consider the initial boundary value problem for the wave equation  $g^{ab}\nabla_a\nabla_b\phi = F$  with boundary condition of the form  $(T^b + aN^b)\nabla_b\phi|_{\mathcal{T}} = q$ ,  $a > 0$ , where  $T^a$  is a timelike unit vector field which is tangent to  $\mathcal{T}$  and  $N^a$  is the outward pointing normal to the boundary  $\partial\Sigma_t = \Sigma_t \cap \mathcal{T}$  in  $\Sigma_t$ . The Sommerfeld condition is given by the choice  $a = 1$ , which corresponds to a condition on the derivative of  $\phi$  in an outgoing null direction. The proof proceeds by considering the fluxes of energy defined with respect to a vector field  $u^a = T^a + \delta N^a$ , where  $\delta > 0$  is a parameter to be chosen. Let  $\Theta^a_b = \nabla^a\phi\nabla_b\phi - \frac{1}{2}\delta^a_b\nabla_c\phi\nabla^c\phi$  be the energy-momentum tensor for the scalar field. The energy  $E$  and outward flux  $\mathcal{F}$  are defined by integration over  $\Sigma_t$  and  $\partial\Sigma_t$ , with respect to the densities  $u^b\Theta^a_b n_a$  and  $u^b\Theta^a_b N_a$ , respectively. Using integration by parts one derives by a straightforward argument an energy estimate bounding  $\partial_t E$  in terms of  $E$ ,  $F$  and  $\mathcal{F}$ . Expanding out the flux density  $u^b\Theta^a_b N_a$  and using the boundary condition to eliminate terms, one finds that for an appropriate choice of  $\delta = \delta(a) > 0$ , it contains a negative multiple of the square gradient of  $\phi$  on  $\partial\Sigma$  plus terms which can be estimated in terms of  $E, F, \mathcal{F}$ . Inserting this in the above mentioned energy estimate yields a form of the energy estimate which gives the boundary regularity required for well-posedness. Based on these ideas, the proof of well-posedness for the initial boundary can now be completed along standard lines.

## 3. MOVING PUNCTURES AND BLACK HOLES

During the time since GRG17, Dublin 2004, there have been tremendous advances in numerical relativity, in particular in the binary black hole problem. The first breakthrough was the work by Frans Pretorius, using a constraint stabilized harmonic coordinate formulation, with excision. Using this code he was during the spring of 2005 able to evolve stably several orbits of a binary black hole system. This was followed shortly thereafter by announcements from several groups using different methods, that they were able to achieve similar results. One of the methods which has emerged as a successful alternative to the harmonic formulation is based on the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation of

the Einstein evolution equations, cf. [49, 7], see also [48]. Due to various issues arising from a combination of several factors (gauge choice, excision etc.), codes based on BSSN initially suffered from instabilities which prevented more than partial orbits to be calculated. However when the proper techniques are applied one may successfully use the BSSN formulation to stably evolve black hole spacetimes, even without excision. The black holes (BHs) are thus included in the domain of computation, and one effectively treats the BHs as essentially weak singularities in the Cauchy slices. This approach is now one of the most popular in the numerical GR community. In spite of the success in using this formulation, the reason why it actually works has remained mysterious.

**Sascha Husa** discussed recent work, cf. [28, 15], which sheds light on this issue. The moving puncture method starts with BHs represented by “filling” their interiors with wormholes, reduced to singular (“puncture”) points by spatial compactification. The moving-puncture method [17, 6] is only a minor modification of previous approaches, in that no attempt is made to factor out the singular puncture geometry during the simulation, but surprisingly in numerical evolutions of a Schwarzschild BH using the BAM code [15], Husa et al. found that this leads to a drastic change in the geometry of the time slices [28]. They have constructed an analytic solution for the stationary state of a nonspinning BH that matches the moving-puncture gauge, and found that the numerical data asymptote to this solution.

The key result about the geometry of moving punctures is that for a Schwarzschild black hole the numerical evolutions approach a stationary slice that neither reaches an internal asymptotically flat end nor hits the physical singularity, as might be expected for a stationary slice with non-negative lapse. Rather, the slice ends at a throat at finite Schwarzschild radius ( $R_0 \approx 1.31M$ ), but infinite proper distance from the apparent horizon. This changes the singularity structure of the “puncture”. It is still a puncture in that there is a coordinate singularity at a single point in the numerical coordinates, but it does not correspond to an asymptotically flat end. In the course of Schwarzschild evolutions Husa et al. have found that the throat does collapse to the origin. Where one would have expected an inner and an outer horizon, they find only one zero in the norm of  $(\frac{\partial}{\partial t})^a$ , corresponding to the outer horizon. An under-resolved region does develop in the spacetime (it is the region between the throat and the interior spacelike infinity), but we are pushed out of causal contact with it. The throat itself has receded to infinite proper distance from the outer horizon. Matter fields or gravitational radiation will be trapped between the inner horizon and the throat, because unlike the gauge their propagation is limited by the speed of light; Husa et al. will consider this issue in future work.

#### 4. ISOLATED SYSTEMS

The analysis of the asymptotic structure of asymptotically flat spacetimes is of central importance in mathematical general relativity. The conformal compactification setting introduced by Penrose allows one to give a stringent analysis of the asymptotic properties of spacetimes, including conserved quantities. Further, the weak cosmic censorship conjecture has a clear formulation in this setting. The work of Klainerman et al. [18, 33] shows that asymptotic fall-off conditions on Cauchy data can be introduced so that the Cauchy development has a conformal compactification with any finite regularity. It is known that there are large classes

of spacetimes which have a conformal compactification which is regular at spatial infinity. It is an interesting open question whether the assumption of smooth null infinity implies some type of rigidity for the spacetime [21, 19]. The regular conformal field equations of Friedrich give a clear cut formulation of this problem, and there is good hope of eventually finding sharp conditions for regularity.

**4.1. Stability of Minkowski space.** The talk of **Lydia Bieri** addressed the global, nonlinear stability of solutions of the Einstein equations in General Relativity. In particular, she discussed results on the initial value problem for the Einstein vacuum equations, which generalizes the results of Christodoulou and Klainerman [18]. Every strongly asymptotically flat, maximal, initial data which is globally close to the trivial data gives rise to a solution which is a complete spacetime tending to the Minkowski spacetime at infinity along any geodesic. In [11], Bieri has considered the Cauchy problem with more general, asymptotically flat initial data. In particular, under the assumptions in [11], the spacetime curvature fails to be continuous. In order to show decay of the spacetime curvature and the corresponding geometrical quantities, the Einstein equations are decomposed with respect to adequate foliations of the spacetime. In the proof of Bieri, one encounters borderline estimates for certain quantities with respect to decay, indicating that the conditions concerning decay at infinity imposed on the initial data are sharp.

**4.2. Rigidity for asymptotically simple spacetimes.** **Juan Antonio Valiente Kroon** reported on progress [53] towards a proof of the following conjecture concerning asymptotically simple spacetimes: *If an asymptotically Euclidean, conformally flat, time symmetric initial data set for the Einstein vacuum equations gives rise to a development admitting a smooth conformal compactification at null infinity, then the initial data must be Schwarzschild in a neighbourhood of infinity.* It should be noted that the Schwarzschild spacetime is the only stationary spacetime admitting conformally flat slices. There are further indications of generalisations of this conjecture for more general classes of data, cf. [54, 55]. A possible proof of the above conjecture requires a detailed understanding of the structure of certain asymptotic expansions of the development of the initial data near null and spatial infinity. The tools to obtain these expansions are the “extended conformal Einstein equations” and the representation of spatial infinity known as “the cylinder at spatial infinity” which have been introduced by Friedrich in [25]. The asymptotic expansions are obtained by solving, for a given initial data set, a hierarchy of interior equations at the cylinder at spatial infinity. These interior equations allows to transport initial data from the initial hypersurface up to the “critical sets” where null infinity “touches” spatial infinity. The structure of the interior equations suggests that their solutions should contain very specific types of logarithmic divergences at the critical sets unless the initial data is Schwarzschild. The explicit existence of these obstructions has been shown in [53].

## 5. SINGULARITIES

**5.1. Stochastic aspects of generic singularities.** The proposal of Belinskii, Khalatnikov and Lifshitz [9, 10, 32] (BKL) on the structure of generic cosmological singularities states roughly that the essential properties of the asymptotic dynamics of the gravitational field along typical timelines can be understood by considering certain spatially homogeneous models. Writing the Einstein equations in terms of

Hubble normalized scale invariant frame variables due to Uggla et al. [52, 1, 47] or the approach of Damour et al. [22], based on Iwasawa decomposition, each of which relies on a long history of previous formulations, gives a description of the asymptotics of the gravitational field in terms of billiard dynamical systems. In the case of the Hubble normalized formulation, one gets a billiard in the Kasner plane, cf. [56], while for the Iwasawa formulation one gets for the case of 3+1 gravity, a billiard in a domain of hyperbolic space, which is analogous to the Misner-Chitre billiard.

**Claes Uggla** presented work [30], building on [52, 1, 47], which generalizes and makes rigorous some aspects of the previous work of BKL and others on the stochastic aspects of the system which arise due to the chaotic nature of the asymptotic billiard systems. Using a combination of dynamical and statistical analyses and in part heuristic arguments, this work describes the generic properties of a “billiard attractor”. It turns out that several degrees of freedom, which a priori could have been of relevance for the asymptotic behavior are, generically, statistically suppressed. The dynamical and statistical arguments of Uggla et al. may be contrasted with results concerning asymptotic behavior in Bianchi type IX obtained using purely dynamical arguments [46]. The results presented by Uggla suggest that the role of Bianchi type IX as a model for the asymptotic dynamics of generic singularities should be re-evaluated, in view of the fact that generic singularities exhibit features that are not shared by Bianchi type IX.

**5.2. Kinematic and Weyl singularities.** For expanding Bianchi cosmologies with a tilted perfect fluid with linear equation of state  $p = (\gamma - 1)\rho$ , all timelike and null geodesics are complete into the future. However, the fluid congruence may be incomplete into the future (called a congruence singularity), accompanied by the blow-up of kinematic quantities associated with the fluid congruence. **Woei-Chet Lim** discussed recent work [20, 37] on the nature of such singularities. Much emphasis has been placed on the blow-up of the components of the Weyl tensor associated with the fluid congruence. However, as shown by examples due to Lim et al., this phenomenon is independent of the incompleteness of the fluid congruence. Hence it is necessary to differentiate congruence singularities into kinematic singularities and Weyl singularities. In particular, the fluid may become extremely tilted or the kinematic or Weyl components may blow up, depending on whether  $\gamma$  exceeds certain critical values.

**5.3. Perturbations of naked singularity spacetimes.** The scalar field may be considered as a toy model for perturbations of a background spacetime. **Brien Nolan** discussed some rigorous mathematical results that probe the linear stability of naked singularities in self-similar collapse. He showed that the multipoles of a minimally coupled massless scalar field propagating on a spherically symmetric self-similar background space-time admitting a naked singularity maintain finite  $H^{1,2}$  norm as they impinge on the Cauchy horizon. Further, each multipole obeys a point-wise bound at the horizon, as does its locally observed energy density [40]. The null energy condition is assumed to hold in the background spacetime. In order to view the scalar field as a toy model for perturbations of a background spacetime, the matter model must be specified. To study such perturbations, the matter content must be specified. Nolan considers what in this context is the simplest case: that of null dust - i.e. the stability of the Cauchy horizon in self-similar

Vaidya space-time is studied. The results for the scalar field carry over to odd-parity linear perturbations at the level of both the metric and the curvature [41]. For even parity perturbations, the linearised Einstein equations can be reduced to a 5-dimensional first order symmetric hyperbolic system. The components of the state vector are gauge invariant metric and matter perturbation quantities. Nolan shows that the  $L^2$  norm of the state vector blows up at the Cauchy horizon and that solutions for which the  $L^\infty$  norm of the state vector does not blow up at the Cauchy horizon are unstable in the space of test function initial data. This provides strong evidence that the Cauchy horizon of the self-similar Vaidya space-time is unstable [39].

**5.4. Spacetime boundaries and properties of singularities.** In his talk, **Benjamin Whale** reported on joint work with Sue Scott, where the a-boundary and the a-boundary singularity theorem are applied to analyze the physical properties of singularities.

## 6. QUASI-LOCAL ASPECTS

**6.1. Towards the quasi-localization of canonical GR.** Perhaps the most natural way of introducing the conserved quantities in asymptotically flat spacetimes is the canonical/Hamiltonian one. A key observation in the Hamiltonian formulation of GR, due to Arnowitt, Deser and Misner [5], is that the evolution parts of Einstein's equations can be recovered formally as canonical equations of motion in the phase space, in which the constraint function (i.e. whose vanishing is just the constraint parts of Einstein's equations) play the role of the Hamiltonian. However, as Regge and Teitelboim stressed, if we want to recover the correct evolution parts of Einstein's equations as partial differential equations for *smooth fields* rather than some distributional generalization of them, then the Hamiltonian in the canonical equations of motion must be *functionally differentiable* with respect to the canonical variables [45]. Regge and Teitelboim showed that such a Hamiltonian can be obtained by adding an appropriate 2-surface integral to the constraint functions. The remarkable property of this Hamiltonian is that the ADM energy-momentum and angular momentum can be recovered as the value of the Hamiltonian on the constraint surface with appropriately chosen lapse and shift. On the other hand, the investigations of Beig and Ó Murchadha showed that for given boundary conditions on the canonical variables the asymptotic form of the lapse and the shift is already determined if we require that the evolution equations preserve the boundary conditions [8]. Moreover, they also showed that both the constraint functions and the Hamiltonians close to Poisson algebras, the former being an ideal in the latter, and their quotient, the so-called algebra of observables, is isomorphic to the Poincare algebra. The ADM conserved quantities then can be considered as appropriate coordinates on this algebra of observables.

**László Szabados** considered in his talk the boundary conditions for canonical vacuum GR at the quasi-local level, i.e. when the spacelike hypersurface on which the canonical variables are defined is compact with smooth 2-boundary  $\mathcal{S}$  [50]. The key ideas used by Szabados are found by analogy with the Hamiltonian analysis in the asymptotically flat case above, namely the requirement of the functional differentiability of all the functions whose Poisson bracket must be calculated, and the requirement of the compatibility of the evolution equations and the boundary

conditions both on the canonical variables and the lapse and the shift. It has been shown by Szabados [51] that fixing the area element on the 2-surface  $\mathcal{S}$  (rather than the whole induced 2-metric) is enough to have a well defined constraint algebra, a well defined Poisson algebra of basic Hamiltonians parameterized by lapses that are vanishing on  $\mathcal{S}$  and shifts that are tangent to and divergence free on  $\mathcal{S}$ . Their quotient is a Lie algebra of a class of 2-surface observables.

The evolution equations preserve these boundary conditions, and the observables (realized as the value of the basic Hamiltonians on the constraint surface) are 2+2-covariant, gauge-invariant and provide a representation of the Lie algebra of the divergence-free vector fields on  $\mathcal{S}$ . Szabados pointed out, that in special, standard situations the observables appear to behave as angular momentum.

**6.2. The Einstein scalar field at finite infinity.** The ‘finite infinity’ paradigm of Ellis was proposed to study isolated gravitational systems, in our universe, where we experience the presence of other local matter fields and a cosmological background [24]. In this context it is not possible to make the assumption of asymptotic flatness at infinity, under which boundary conditions on matter have been studied in detail. Instead, a smooth timelike surface  $\mathcal{F}$ , is introduced. The timelike surface  $\mathcal{F}$  should be located at a large finite distance from the centre of the local system of interest, with the aim to study the fields generated from this on  $\mathcal{F}$ .

**William Clavering** in his talk examined the behaviour of the spherically symmetric Einstein scalar field at  $\mathcal{F}$ . He considered the evolution of a scalar field over a domain bounded by an initial hypersurface, characteristic curves, and  $\mathcal{F}$ . Using Hawking’s mass formula [29], he has studied the consequences of imposing conditions on the mass-energy flux at  $\mathcal{F}$ . Two examples were discussed; each a perturbation of a static exact solution. For a perturbation of Schwarzschild there is no mass-energy flux of the scalar field. Clavering conjectures that this is the case in all vacuum spacetimes. The analysis has been repeated for the pure scalar field case of the Wyman spacetime [57]. In this case, non-trivial expressions for the mass-energy flux in terms of the scalar field at  $\mathcal{F}$  were obtained.

**6.3. Quasi-local energy inside the event horizon.** Pointlike objects cause many of the divergences that afflict physical theories. For instance, the gravitational binding energy of a point particle in Newtonian mechanics is infinite. In general relativity, the analog of a point particle is a black hole and the notion of binding energy must be replaced by quasilocal energy. The quasilocal energy (QLE) derived by York, and elaborated by Brown and York [13], is finite outside the horizon but it was not considered how to evaluate it inside the horizon.

**Andrew Lundgren** presented a prescription for finding the QLE inside a horizon, and showed that it is finite at the singularity for a variety of types of black hole. It turns out that the energy is typically concentrated just inside the horizon, not at the central singularity. It is impossible to localize gravitational energy due to the equivalence principle, so it is meaningless to define gravitational energy at a single point. This problem is avoided by considering the gravitational energy in a region of space. The boundary of the region is a two-dimensional surface, and in fact the quasilocal energy is defined only in terms of quantities defined on the enclosing surface (specifically the induced metric and extrinsic curvature). The QLE is defined only in terms of the gravitational degrees of freedom and makes

no mention of any other fields that are present. However, since gravity couples to everything, the QLE counts all energy that is present in the region.

Lundgren et al. [38] have considered only spherically-symmetric static black holes. The definition of the QLE in [13] was extended to be valid inside the event horizon, which is only a coordinate singularity. In the Schwarzschild case, the QLE at the central singularity is zero. The analogous quantity for a classical field of a point particle diverges at the center. The gravitational field itself carries energy which gravitates, causing the nonlinearity of general relativity. Lundgren remarked that the nonlinearity somehow provides a mechanism that cures the divergence expected of a point particle field. The definition of the QLE presented by Lundgren can be applied also to the deSitter case and the case with non-vanishing charge.

## 7. BLACK HOLES

### 7.1. Black hole rigidity and spacetimes with compact Cauchy horizon.

**Jim Isenberg** reported on recent work with Vince Moncrief which studies analytic solutions of Einstein's equations containing nondegenerate compact Cauchy horizons with non closed generators. If certain hypotheses are added, the spacetimes necessarily admit an isometry which acts along the generators of the horizon. Isenberg showed how to use these results to prove that stationary (non static) analytic black holes in arbitrary dimensions necessarily admit symmetries which are independent of the presumed stationarity symmetry. This work is closely related to work of Hollands et al. [31].

### 7.2. Electromagnetic field on the background of high dimensional black holes.

In the last years various generalizations of black hole solutions to the high dimensional gravity have been found. One of them describes a generally rotating black hole with NUT charges. This solution possesses several nice properties as, e.g., the existence of the Killing-Yano tensor, the complete integrability of the geodesic motion or the separability of the Hamilton-Jacobi and Klein-Gordon equations. Its generalization including the acceleration of the black holes or the electromagnetic field is not, however, straightforward.

In four dimensions it is possible to generalize the black hole solution to describe the accelerated black holes (the Plebański-Demiński solution). This transition is achieved by the conformal rescaling of the metric accompanied by a modification of some metric functions. **Pavel Krtoš** showed in his talk that such a procedure is not sufficiently general in higher dimensions—only the maximally symmetric spacetimes in ‘accelerated’ coordinates are obtained. Further, he presented general algebraically special solutions of the Maxwell equations on the background of the mentioned high dimensional generally rotating black hole. They are adjusted to the algebraic structure of the background which is reflected by the existence of the principal Killing-Yano tensor. Such electromagnetic fields generalize the field of charged black holes in four dimensions. However, one may show that in higher dimensions such fields cannot be easily modified in such a way that they would satisfy full Maxwell-Einstein equations.

## 8. POST-NEWTONIAN EXPANSIONS

Post-Newtonian expansions (PNEs) are expansions of the Einstein equations in the parameter  $1/c$ , around  $c = \infty$ , where  $c$  is lightspeed. The limit  $c = \infty$  is the



Newtonian limit of general relativity. Such expansions have been studied since the discovery of general relativity and there is a large literature available. Recently, post-Newtonian expansions have been used to calculate wave forms for the gravitational wave emissions from binary black hole inspirals. These have been compared with numerically calculated wave forms and turn out to be highly accurate until the last stage of the inspiral.

The majority of results on PNEs are based on formal expansions which are used to calculate approximate values of physical quantities, see eg. [23, 12] and references therein. However, this formal approach does not deal with the question of convergence. In the absence of a precise notion of convergence, it becomes unclear to what extent the formal expansions actually approximate relativistic solutions. In view of the importance of the applications of PNEs, it is interesting to give a rigorous foundation to the procedure.

**Todd Oliynyk** discussed his recent work on post-Newtonian expansions for the Einstein-Euler equations. Oliynyk considers expansions of solutions to these equations in the parameter  $\epsilon = v_T/c$  about  $\epsilon = 0$ , where  $v_T$  is a characteristic velocity scale associated with the fluid matter. By rescaling the gravitational and matter variables, the Einstein-Euler equations can be written as

$$(3) \quad G^{ij} = 2\kappa\epsilon^4 T^{ij} \quad \text{and} \quad \nabla_i T^{ij} = 0$$

where  $\kappa = 4\pi G\rho_T/v_T^2$ ,  $v_i v^i = -\epsilon^{-2}$ ,  $\rho_T$  is a characteristic value for the fluid density, and  $t = x^4/v_T$  is a “Newtonian” time coordinate. In his talk, Oliynyk presented results on PNEs for a class of polytropes, which go beyond formal considerations [43, 44]. By introducing suitable renormalized variables for which the limit  $\epsilon \rightarrow 0$  makes sense, and introducing a suitable gauge, Oliynyk derives a family of symmetric hyperbolic systems, depending on the parameter  $\epsilon$ . This system is studied in a class of weighted Sobolev spaces  $H_{\delta,\epsilon}^k$ , which interpolate between the weighted spaces  $H_\delta^k$  and the standard spaces  $H^k$ . Using these spaces, it is possible to prove  $\epsilon$ -dependent energy estimates for solutions to the Einstein-Euler equations. These estimates are then used to prove the existence of convergent expansions in  $\epsilon$  for suitably chosen initial data. Oliynyk discussed the relation between his convergent expansions and the formal PNEs. In order to recover the PNEs to a certain order requires the initial data to be chosen correctly. The method used to choose initial data is based on ideas in [14]. Oliynyk discussed how to recover the post-Newtonian expansion to 2<sup>nd</sup> order in his framework.

## 9. MISCELLANEOUS

**Wave tails in curved spacetimes.** **Risto Tammelo** reported on joint work with Laas and Mankinen dealing with the wave tails of scalar and electromagnetic fields. Their work shows that the intensity of the tail of the electromagnetic wave pulse emitted by a wave source within a compact binary in the vicinity of the geometric shadow of the source of gravitation can be of the same magnitude as the intensity of the direct pulse. The energy carried away by the tail can amount to approximately 10% of the energy of the low-frequency modes of the direct pulse. As an example of an exactly solvable model system, a scalar wave field on a particular Robertson-Walker spacetime has been considered. In the case of minimal coupling, if the metric describes the Friedman dust-dominated universe, the higher-order multipole solutions do not contain a wave tail term. A massless nonconformally-coupled scalar

field is also considered in a class of Robertson-Walker spacetimes. Tammelo et al. show that an initially massless scalar field in the Robertson-Walker universe can obtain a mass in the corresponding asymptotic Minkowski space region.

**Ricci flow and the Einstein equations.** Eric Woolgar described work on the Ricci flow of asymptotically flat manifolds in the rotationally symmetric case [42]. This paper shows that if no minimal hypersphere is present initially, then one never forms. The flow then exists for all future time and converges to flat spacetime in the limit of infinite time, the limit being taken in the Cheeger-Gromov sense. The mass does not change during the flow, but the quasilocal mass within any fixed hypersphere about the origin (defined either by fixing the proper radius, the surface area, or the enclosed volume) evaporates away smoothly. **Mohammad Akbar** discussed relations between Ricci solitons and solutions to the Einstein-scalar field equations.

**Posters.** Several interesting abstracts had to be relegated to the poster session. Among these were abstracts by **Håkan Andréasson** on the formation of black holes in spherically symmetric gravitational collapse [4], **Jim Isenberg** on the AVTD behavior of  $T^2$  symmetric solutions of the Einstein vacuum equations, **Makoto Narita** on cylindrically symmetric expanding spacetimes, **Oscar Reula**, [34], showing that the Robinson Trautman Maxwell equations do not constitute a well posed initial value problem, and **Juan Antonio Valiente Kroon** on a characterization of Schwarzschild initial data sets [26].

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